

Mini Golf



Can you get a hole-in-one?

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Objective: Find a path for the ball to follow to make a hole-in one using reflections to find angles of incidence equal to angles of reflection. Explore the properties of the path and other applications related to this problem.

Materials: You'll need paper, tape, a protractor straightedge, a mira (sometimes called a geo reflector, geo reflecta, or something similar), and colored pencils.

Mathematical Practices: I think you could easily argue that all of the CCSSM practices are exhibited in this lesson!

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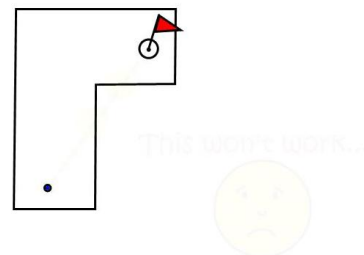
Did you know?

In the 1930's there were **30,000** miniature golf courses and **150** were roof top courses in New York City



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Try a direct shot



There is a control at the bottom of this page. To show the animation press the arrow and show that a direct shot is NOT possible on this hole without bouncing it off of at least one wall!

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Let's try to

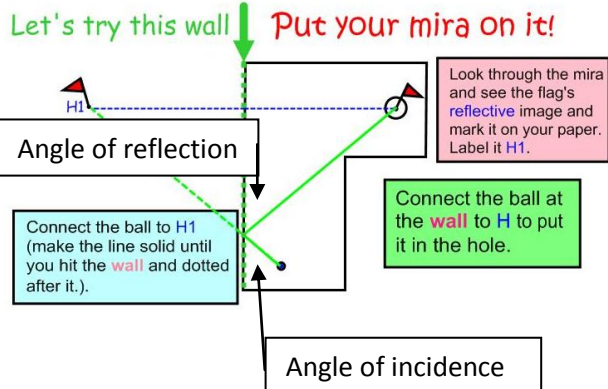


Bank off of one wall

Banking the ball off of one wall – make sure to have a large enough piece of paper to construct/find the points of reflection --- be ready to tape another piece of paper to the original one if you need it!

1. Locate the hole (H) & beginning position of the ball (B).
2. Pick a wall to bank the ball off of and label it W1.
3. With a compass or mira, reflect the hole (H) over wall 1 and mark it H1.
4. With a straight edge, connect the ball (B) with H1 using a dotted line to locate where on the wall the ball needs to reflect off of.

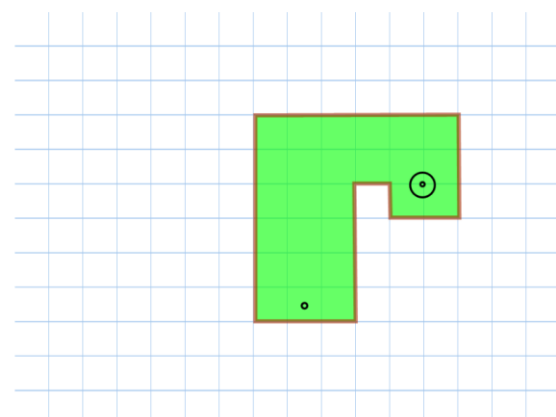
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5. Connect the hole (H) to the point of intersection of the wall and the line from the ball to H1 to complete the path of the ball. The solid lines show the path the ball will take to make the hole in one. Realize that speed of the ball, the type of surface used on the hole, the accuracy of the golfer, and other factors may influence the actual success of the hole-in one! (See the diagrams below showing two examples of the same golf hole but banking the ball off of different walls.)

NOTE: This is close to the corner, but if you are a good golfer, you can get a hole-in-one. ☺

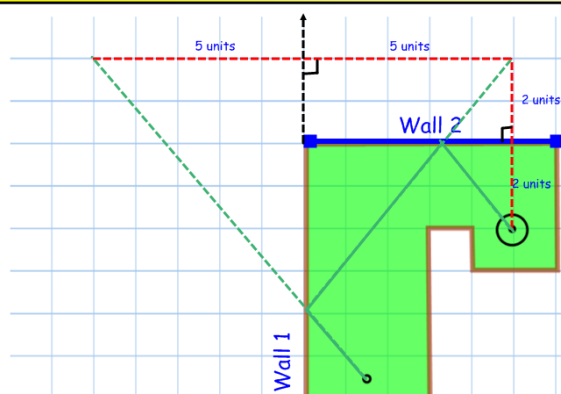
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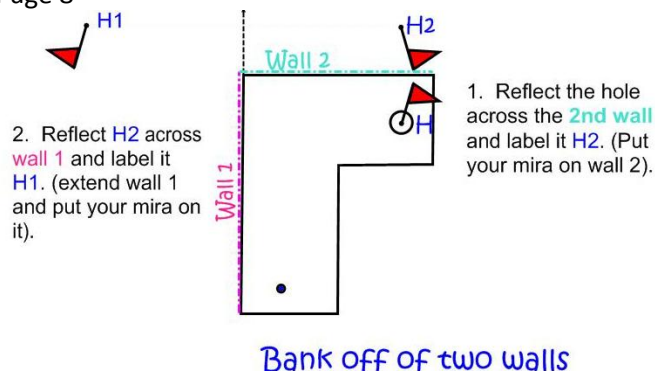
Let's try finding a path first on a hole plotted on graph paper...

1. Locate the hole (H) & beginning position of the ball (B).
2. Pick and label the walls starting at the hole and working backwards from the hole. Mark the walls W2 for the second wall the ball will hit before landing in the hole, and W1 for the first wall to bank it off to send it to the second wall. (in this case the first wall hit by the ball).
3. Using the squares on the graph paper and properties of a reflection, reflect the hole (H) over the wall 2 (W2 on this example) and mark it H2 by counting squares as shown in the second diagram to the left.
4. Now, reflect H2 over wall 1 (W1) and mark it H1. You may have to extend a wall to be able to complete a reflection. Use the squares on the graph paper to locate the H1.
5. With a straight edge, connect the ball (B) with H1 to locate where on the wall the ball needs to hit. Then connect that point of contact/intersection with H2 to locate where on wall 2 the ball needs to hit.
6. Finally, connect the point of contact/intersection on the wall 2 with the hole (H) to complete the path of the ball. (See the diagram.)
7. Notice that at each point of contact with the wall the angle of incidence = angle of reflection! (BUT, don't make the mistake that ALL of the angles at those points are NOT the same measures!)

Let's use graph paper to learn to bank a ball off of two walls...

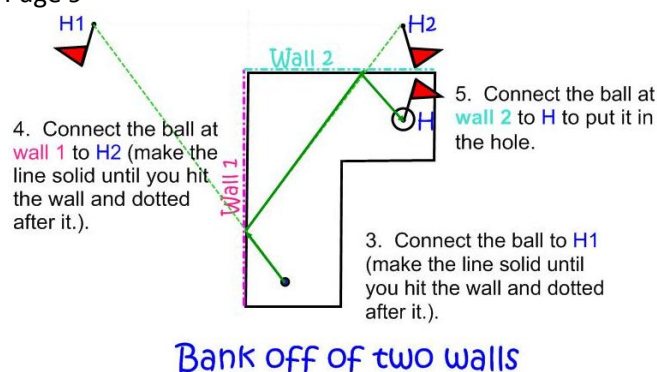


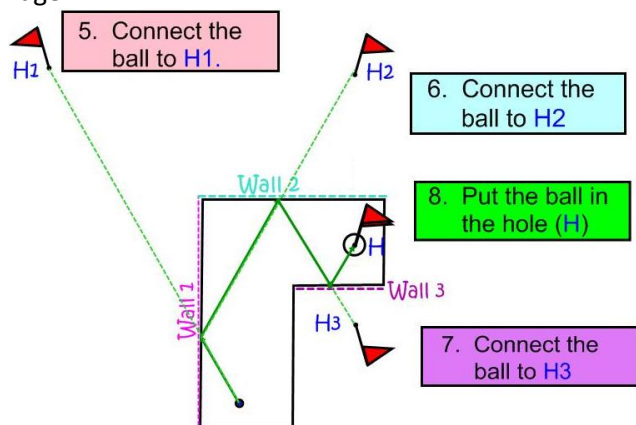
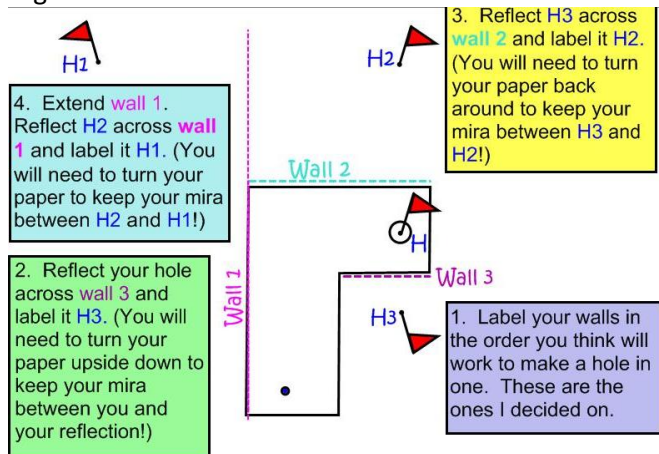
(*Note: not all choices of walls will result in possible holes-in-one because of the design of the hole. If you begin connecting points and the points of contact do not end up on the correct wall, that path is not possible and other walls should be chosen or the placement of the ball or the hole must be changed.)



This animation to bank the ball off of two walls to make a hole-in-one is shown over two pages in SMART Notebook.

1. Locate the hole (H) & beginning position of the ball (B).
2. Pick and label the walls starting at the hole and working backwards from the hole. Mark the walls W2 for the second wall the ball will hit before landing in the hole, and W1 for the first wall to bank it off to send it to the second wall. (in this case the first wall hit by the ball).
3. With a compass or mira, reflect the hole (H) over the wall 2 (W2 on this example) and mark it H2. You may have to extend a wall to be able to complete a reflection.
4. With a compass or mira, reflect H2 over wall 1 (W1) and mark it H1.
5. With a straight edge, connect the ball (B) with H1 to locate where on the wall the ball needs to hit. Then connect that point of contact/intersection with H2 to locate where on wall 2 the ball needs to hit.
6. Finally, connect the point of contact/intersection on the wall 2 with the hole (H) to complete the path of the ball. (See the diagram.)
7. Notice that at each point of contact with the wall the angle of incidence = angle of reflection! (BUT, don't make the mistake that ALL of the angles at those points are NOT the same measures!)





This animation to bank the ball off of three walls to make a hole-in-one is shown over two pages in SMART Notebook (pages 11-12).

Banking the ball off of 3 or more walls

(*Note: not all choices of walls will result in possible holes-in-one because of the design of the hole. If you begin connecting points and the points of contact do not end up on a wall, that path is not possible and other walls should be chosen or the placement of the ball or the hole must be changed.)

1. Locate the hole (H) & beginning position of the ball (B).
2. Pick and label the walls starting at the hole and working backwards from the hole. Mark the walls W1 for the first wall the ball will hit before landing in the hole, W2 for the next wall, W3 for the next wall to hit, etc.
3. With a compass or mira, reflect the hole (H) over wall with the highest number and mark it H# (this example: H3 if you going to bank it off of 3 walls, H4 if you're planning to hit 4 walls, etc.). You may have to extend a wall to be able to complete a reflection.
1. With a compass or mira, reflect H' over wall 2 (W2) and mark it H'2. Then reflect H2 over wall 1 (W1) and label it H1.
4. With a straight edge, connect the ball (B) with H1 to locate where on the wall the ball needs to hit. Then connect that point of contact/intersection with H2 to locate where on wall 2 the ball needs to hit. Then continue until all of the walls chosen have points of contact/intesection.
5. Finally, connect the point of contact on the last wall with the hole (H) to complete the path of the ball. (See the diagram.)
6. Notice that at each point of contact with the wall the angle of incidence = angle of reflection! (BUT, don't make the mistake that ALL of the angles at those points are NOT the same measures!)

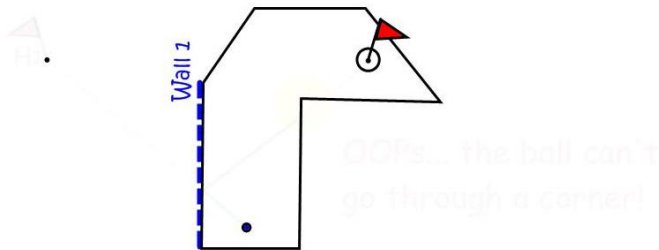
NOTE: If you want to place obstacles on the hole, do it AFTER you find the path of the ball to make sure that the obstacle doesn't get in the way of the ball and the path it needs to follow.

This diagram shows how to make a hole-in-one by banking the ball off of 5 walls. Notice that at each point of contact with the wall the angle of incidence = angle of reflection! (BUT, don't make the mistake that ALL of the angles at those points are NOT the same measures!)



Even though the next two holes have had their reflections done correctly, they do not produce paths that work to get holes-in-one...

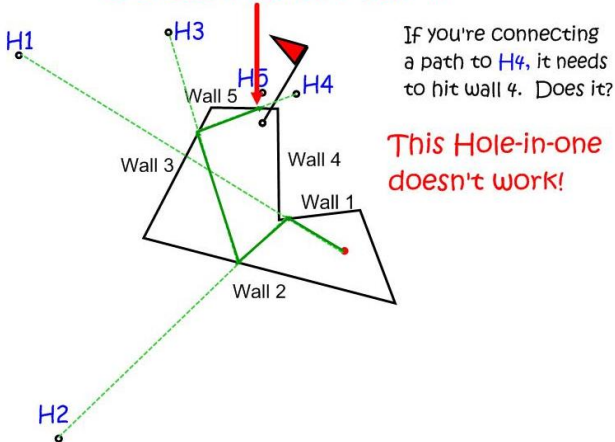
Bank off of one wall



The path still needs to go through a wall, therefore it does not work!

This Hole-in-one doesn't work!

OOPs.... this isn't wall 4!



Notice that when connecting this path, it is important to make sure that the path to H4 hits wall 4 and not another wall. NOTATION and LABELING is crucial to constructing this hole-in-one to make sure it is going to work!!

Student Projects

Golf Course A
Golf Course BGolf Course C
Golf Course D

There are pull tabs on this page to show examples of student made golf courses. Pull them out to show examples!



1. When banking off of certain walls, is there more than one path to bank it off the walls in the same order? How do you know?

The answer is no... but you can construct the path by starting by reflecting the ball and reversing the process... make sure to connect from the hole back in this case!



2. It has been conjectured that the ball that bounces off the walls and goes in the hole follows the shortest path that accomplishes this task.

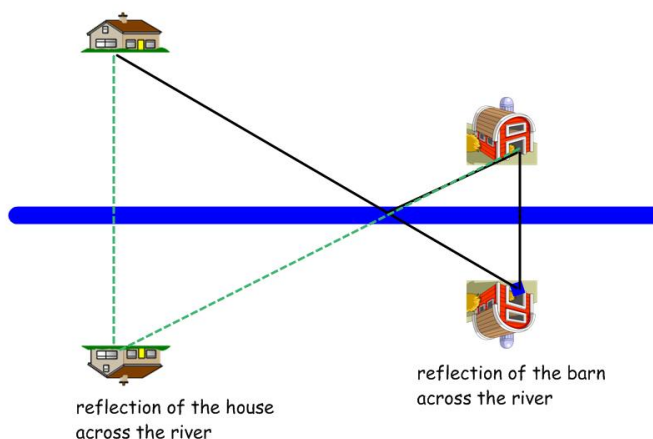
- True or false?
- Prove your answer.

This is true. Move the ball slightly along the wall and draw in triangles. The secret is in showing that the shortest distance between two points is a straight line and this can be proven using the Triangle Inequality theorem.

3. A farmer needs to run a pipeline from the river to both his home and his barn (as pictured) to pump water from the river. Where would place the pump on the river's edge to ensure the least cost when purchasing pipe? Is there more than one place to put the pump? How do you know it is the shortest... prove it! If one cm on our drawing = 1.5 yard, how long does the pipe need to be?



Solution for Problem 3



This problem is really the golf problem in disguise with the river being a wall and the house and barn could easily be the hole and the ball. This problem is used to minimize distances for laying pipe. This could also be used to explore electricity and the placement of a transformer.

The second part of the problem is focusing on the properties of similar figures/scale drawings.

Resources and References:

1. Powell, Nancy Norem, Mark Anderson & Stanley Winterroth, *Reflections on Miniature Golf*, The Mathematics Teacher, Oct 1994, 490-495
2. Directory of 37 Online Miniature Golf Games:
<http://www.thepinballzone.net/games/online-mini-golf/>
3. Can you make a hole-in-one?
<http://www.figurethis.org/challenges/c40/challenge.htm>
4. Mirror, Mirror: Reflections and Congruencies
<http://www.andrews.edu/~calkins/math/webtexts/geom04.htm>
5. Mini golf activity <http://www.csiro.au/scope/activities/e29c02activity.htm>
6. Laws of Reflection <http://www.worsleyschool.net/science/files/amusement/reflection.html>
7. NCTM Illuminations:
 - a. Paper Pool applet: <http://illuminations.nctm.org/ActivityDetail.aspx?id=28>
 - b. Analyzing Numeric and Geometric Patterns of Paper Pool:
<http://illuminations.nctm.org/LessonDetail.aspx?ID=L244>
8. A Mini-Golf Hole So Real It's Unreal: Intense reflection wins NYC art design contest for UB architect Joyce Hwang <http://www.buffalo.edu/news/9510>
9. Miniature golf books and articles <http://www.miniaturegolfer.com/minigolfbooks.html>
10. Hole-in-One! Activitiy
http://online.math.uh.edu/MiddleSchool/Modules/Module_4_Geometry_Spatial/Activities/Hole_One/UHGeometryHoleinOneTeacherNotes.pdf
11. Math in your life – golf (video) <http://www.youtube.com/watch?v=26HqsQEmV-8>
12. Mathematics that Swings: the Math Behind Golf: lecture by Dr. Douglas N. Arnold (video – warning.. this is just a little over 1 hour long!) <http://www.youtube.com/watch?v=e6v9ib-dOtg>

